Decay of the false vacuum through dissipative tunneling

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The effect of dissipation on tunneling is investigated by solving numerically the time-dependent Schrödinger equation for a double-well potential both in the resonant and the nonresonant case. Two existing dissipative terms are studied and an additional term appropriate for the damping of the squeezing motion is proposed.

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I. INTRODUCTION

A false vacuum decays due to thermal fluctuations or through quantum tunneling. This process is the heart of the reaction rate theory and was intensively investigated in connection with the nucleation phenomena in statistical physics [1], with fission decay in nuclear physics [2], and with inflationary models of the universe in field theory (cosmology) [3,4].

The equilibrium rate of the thermally activated nucleation, such as the condensation of a supersaturated vapor or the boiling of a superheated fluid may be expressed as $\lambda_R = \nu e^{-E_b/kT}$, where E_b is the activation energy and ν a model-dependent factor. The stationary rate for decay through quantum tunneling of a metastable state was calculated using various semiclassical methods (WKB, path integrals) as $\lambda_{st} = f e^{-W_b/\hbar}$, where f is the frequency of "barrier assaults" and W_b is the effective action evaluated along the classical periodic orbit through the turning points in the inverted potential [5,6].

The importance of dissipation in the computation of ν became first evident in the chemical reaction rate theory [7]. These reactions are activated by the molecular Brownian motion induced by the thermal random forces connected through the fluctuation-dissipation theorem with the temperature T and the friction coefficient γ. Thus, we may expect a strong decrease in the reaction rate both at weak and strong friction. For the case of moderate-to-strong friction, the reaction rate is given by Kramers [8]

$$\lambda_R = \frac{\omega_a}{2\pi}(\sqrt{1+\eta_b^2} - \eta_b) e^{-E_b/kT}, \tag{1}$$

where $\eta_b = \gamma/2\omega_b$, and $\omega_{a(b)}$ are the frequencies of the linearized molecular potential around the metastable minimum (a) and around the barrier (b).

The effect of dissipation on bound quantum states was a puzzling problem since the early days of the quantum theory, when it was realized that the atomic electrons do not dissipate the energy by electromagnetic radiation as was predicted by the classical electrodynamics. For the first tunneling process investigated, the α decay, the dissipation was also practically absent, and the decay rates were described satisfactorily by the Gamow formula. The importance of dissipation during nuclear decay was mentioned for the first time in connection with the near-barrier fission of actinide nuclei [8]. Although initially there seemed to be no need for dissipation, because the fission widths were well reproduced by the simple Bohr-Wheeler formula [9], the calculus of the kinetic energy in the final state suggested the opposite. At normal fission, not all the reaction energy is released as kinetic energy of the fragments, but about 10% is transferred to their intrinsic degrees of freedom during the descent from the saddle point to the scission configuration [10]. In principle, this transfer could be accounted for if the cross section was obtained by a coupled channel calculation including many inelastic channels. Since such a calculation is not feasible, it appears natural to model the global effect by a phenomenological friction coefficient for the nuclear shape dynamics. Unfortunately, even at this level the problem has no definite answer, because the type of the nuclear friction and the related dissipation mechanism are not yet known. However, assuming friction forces linear in the velocities and high excitation energies, the Kramers formula can be directly applied to estimate the fission decay rate [11].

Attempts to develop a quantum theory of dissipative tunneling are relatively recent, starting with the work of Caldeira and Leggett [12]. Their result was that for potentials with a single metastable minimum, at $T=0$ the dissipation decreases the tunneling rate exponentially.

For double-well potentials new effects occur due to back scattering. Without dissipation, in such systems the phenomenon of "quantum coherence" appears, when the localization probability in either well undergoes periodic oscillations. In general, these oscillations characterize two-level systems [13] and may be observed also for nongeometrical degrees of freedom, as is the case of the neutrino oscillations or the strangeness oscillations of the K^0 mesons [14]. Within the two-level approximation, it was shown [15] that in symmetric double-well potentials the presence of the frictional forces destroys the quantum coherence oscillations and the tunneling becomes aperiodic. Moreover, when the friction coefficient

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exceeds the critical value $\gamma_c = 2\pi\hbar/mD^2$, with m the particle mass and D the distance between the potential minima, the "self-trapping" may appear, namely, after infinite time the particle has more than a 50% probability to be localized in the initial well.

For asymmetrical double-well potentials, the problem of self-trapping and energy loss was considered in [16], showing that the dissipated energy saturates at a value limited by the bottom of the stable well and the particle is always allowed to escape. Thus, the self-trapping does not appear in this case, although the tunneling is strongly suppressed when the friction constant exceeds γ_c. However, more exact calculations of the tunneling probability using the time-dependent Schrödinger equation (TDSE) have shown that in such double-well potentials the resonance effects are very important, and the nonresonant states practically do not tunnel even in the absence of dissipation [17]. In addition, saturation value of the energy loss should be clearly limited by the true ground-state energy.

The purpose of the present work is to clarify these aspects by investigating the tunneling for resonant and nonresonant states in double-well potentials using TDSE with dissipative terms. Because there is no standard way to introduce dissipative terms in TDSE, we consider two phenomenological models, presented in Sec. II. In the first, the rate of energy decrease is supposed to be proportional to its fluctuation (Gisin [18]), while in the second it is proportional to the square of the average momentum (Albrecht [19]). The Gisin model, being a special case of the optical model, is expected to be relevant when both the particle and the environment are quantum objects. In the semiclassical limit, when the frictional force tends to be proportional to the velocity, the Albrecht model should apply.

Before investigating the tunneling problem, Sec. III discusses the effect of the Gisin and Albrecht terms on the time evolution of the Gaussian wave packets in a harmonic oscillator potential. The main difference between these terms is revealed by the "squeezing" motion of the wave function [20], which is not damped by the linear friction term. The Albrecht-like dissipative term for squeezing will be constructed by using appropriate coordinate and momentum operators.

Section IV presents numerical results for dissipative tunneling of a Gaussian wave packet in an asymmetric double-well potential with a variable width of the stable well. By changing this width, it is possible to study the resonance effects. Section V is devoted to a survey of the main results and to the concluding remarks.

II. PHENOMENOLOGICAL MODELS FOR DISSIPATION

Within the phenomenological models one assumes that the pure states are preserved during the time evolution and the "free" Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = H_0\psi \tag{2}$$

is modified due to the coupling with the environment by a term $W_{(t)}$, additional to H_0, accounting for the loss of energy. This term depends on ψ, making the modified equation nonlinear and violating the superposition principle.

Although there is rich literature on the possible dissipative terms, in the following we will restrict ourselves essentially to two models [18,19], which we believe to be representative.

A. Gisin model

This approach [18] follows the lines of the optical model where the inelastic channels are accounted for by the use of a complex potential. Additional complex terms were used for the treatment of a large variety of phenomena in nuclear [21] as well as in solid-state physics [22]. A phenomenological treatment of dissipative tunneling for squeezed states in double-well potentials by adding imaginary terms to the energy was given in [23]. In Gisin's model, they are constructed by using the initial Hamiltonian, although this choice cannot be related in general to the velocity-dependent frictional force from the classical mechanics.

Within the Gisin model, the time evolution of the wave function $|\psi\rangle$ is given by

$$i\hbar\frac{\partial\psi}{\partial t} = H_0\psi - i\lambda(H_0 - \langle\psi|H_0|\psi\rangle)\psi \ , \tag{3}$$

where λ is the damping coefficient. The rate of dissipation for the energy $E = \langle\psi|H_0|\psi\rangle$ is

$$\frac{dE}{dt} = -\frac{2\lambda}{\hbar}(\langle\psi|H_0^2|\psi\rangle - \langle\psi|H_0|\psi\rangle^2) \ , \tag{4}$$

appearing proportional to its fluctuation. Therefore, the eigenstates of H_0 remain stationary, and there is no dissipation for the free particle. Otherwise the energy is dissipated until the wave function approaches the state with minimum energy contained in the initial wave packet [18].

B. Albrecht model

In this model [19] the additional term $W_{(t)}$ is chosen such that when $H_0 = p^2/2m + V_{(x)}$, the correspondence principle is fulfilled and the Ehrenfest theorem gives for the expected values of the coordinate and momentum the classical equations of motion with friction

$$\frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m} \ , \tag{5}$$

$$\frac{d\langle p\rangle}{dt} = -\langle\frac{\partial V}{\partial x}\rangle - \gamma\langle p\rangle \ . \tag{6}$$

These equations are not very restrictive for the choice of $W_{(t)}$, and additional criteria may be added, namely that the ground state of H_0 is preserved and the energy dissipation law has the classical form $\dot{E} = -\gamma/m\langle p\rangle^2$. Thus, a free particle dissipates energy, while a stationary state of vanishing average momentum remains stationary. This behavior seems similar to the one required by the virial theorem in the classical mechanics [24], where for a system of particles subject to conservative forces and frictional forces proportional to the velocity the motion does not die, but it reaches a stationary state.

An operator satisfying these conditions and which is invariant to the space translations is $W_A=\gamma(x-\langle x\rangle)\langle p\rangle$, with γ the friction constant. Thus, the modified TDSE is

$$i\hbar\frac{\partial\psi}{\partial t}=H_0\psi+\gamma(x-\langle\psi|x|\psi\rangle)\langle\psi|p|\psi\rangle\psi \tag{7}$$

and the corresponding dissipation law

$$\frac{d\langle\psi|H_0|\psi\rangle}{dt}=-\frac{\gamma}{m}\langle\psi|p|\psi\rangle^2 . \tag{8}$$

However, in the present study we will use for simplicity the "noninvariant" form $W_A=\gamma x\langle p\rangle$, because the time-dependent function $-\gamma\langle x\rangle\langle p\rangle$ contributes to the solution of TDSE only with a coordinate independent phase factor.

Besides the arguments based on the correspondence principle, it is possible to derive the Albrecht term by a variational treatment of the standard model for dissipative systems, assuming the quantum particle to be coupled bilinearly to a bath of classical harmonic oscillators (see the Appendix). The variational approach has the quality of avoiding the ambiguities related to the canonical quantization of the Langevin equation [25,26], and in general it produces a nonlinear Schrödinger equation with memory friction.

III. GAUSSIAN WAVE PACKETS IN A HARMONIC OSCILLATOR POTENTIAL: ANALYTICAL RESULTS

A straightforward application of the phenomenological models is the time evolution of the Gaussian wave packets by the harmonic oscillator Hamiltonian

$$H_0=\frac{p^2}{2m}+\frac{m\omega_0^2}{2}x^2 . \tag{9}$$

Let us consider at $t=0$ a Gaussian wave packet

$$\psi(x,t=0)=\left[\frac{c_0}{\pi}\right]^{1/4}e^{-c_0(x-u_0)^2/2} \tag{10}$$

having the width $\sigma_0^2=\langle\psi|x^2|\psi\rangle-\langle\psi|x|\psi\rangle^2=1/2c_0$, placed at $\langle\psi|x|\psi\rangle=u_0$, and with no momentum, $\langle\psi|p|\psi\rangle=v_0=0$. A trivial situation appears when $c_0=c_p$, $c_p=m\omega_0/\hbar$, and $u_0=0$, because in this case the considered wave packet is the exact ground state $|g\rangle$ of H_0 and the whole evolution is given by a time-dependent phase factor $e^{-i\omega_0 t/2}$.

A. Coherent states

If $c_0=c_p$ but $u_0\neq 0$, the wave packet (10) is a "shifted ground state" $|\psi_G\rangle$ because it is eigenstate of the shifted Hamiltonian $H_{u_0}=p^2/2m+m\omega_0^2(x-u_0)^2/2$ and is related to the ground state of H_0 by a unitary transformation generated by the momentum operator p

$$|\psi_G\rangle=e^{-(iu_0/\hbar)p}|g\rangle . \tag{11}$$

This state is nonstationary and is called the Glauber coherent state. Without dissipation it preserves its shape in time, but it changes the average position $u_t=\langle\psi_G(t)|x|\psi_G(t)\rangle$ and the average momentum $v_t=\langle\psi_G(t)|p|\psi_G(t)\rangle$ according to the classical Hamilton equations for the harmonic oscillator, such that $u_t=u_0\cos(\omega_0 t)$, and $v_t=-m\omega_0 u_0\sin(\omega_0 t)$. In the coordinate representation, the wave packet at the time t, $|\psi_G(t)\rangle=e^{-(i/\hbar)H_0 t}|\psi_G(0)\rangle$ is

$$\psi_G(x,t)=\left[\frac{c_0}{\pi}\right]^{1/4}e^{-i\omega_0 t/2}e^{-c_0(x-u_t)^2/2+(i/\hbar)v_t(x-u_t/2)} . \tag{12}$$

This wave packet may be written also as

$$|\psi_G(x,t)\rangle=e^{-i\omega_0 t/2}e^{z_t b^+-z_t^* b}|g\rangle . \tag{13}$$

Here $z_t=\sqrt{c_p/2}(u_t+iv_t/\hbar c_p)=e^{-i\omega_0 t}z_0$ is a complex function of time, and $b=\sqrt{c_p/2}(x+ip/\hbar c_p)$ is the Dirac-Fock annihilation operator.

The addition of the dissipative terms $W_G=-i\lambda(H_0-\langle H_0\rangle)$ or $W_A=\gamma x\langle p\rangle$ preserves this form of $|\psi_G(x,t)\rangle$, (up to a time-dependent phase factor) and, therefore, the Glauber states remain coherent. The effect of the frictional forces appears only on the time evolution of the parameters u and v, which now, instead of the harmonic evolution have damped oscillations.

In the case of W_G, the evolution of the Glauber wave packet is given by

$$|\psi_G(x,t,\lambda)\rangle=e^{-i\omega_0 t/2}e^{z_t^\lambda b^+-(z_t^\lambda)^* b}|g\rangle , \tag{14}$$

with $z_t^\lambda=e^{-\lambda\omega_0 t}e^{-i\omega_0 t}z_0$ and the average position and momentum:

$$u_t^\lambda=u_0 e^{-\lambda\omega_0 t}\cos(\omega_0 t),\quad v_t^\lambda=-m\omega_0 u_0 e^{-\lambda\omega_0 t}\sin(\omega_0 t) .$$

If the additional term is W_A, the wave packet becomes

$$|\psi_G(x,t,\gamma)\rangle=e^{-i\Phi(t)}e^{z_t^\gamma b^+-(z_t^\gamma)^* b}|g\rangle , \tag{15}$$

with $z_t^\gamma=\sqrt{c_p/2}(u_t^\gamma+iv_t^\gamma/\hbar c_p)$, $u_t^\gamma=u_0 e^{-\gamma t/2}\cos(\Omega t+\phi_0)/\cos\phi_0$, $v_t^\gamma=-m\Omega u_0 e^{-\gamma t/2}\sin(\Omega t)/\cos^2(\phi_0)$, $\tan(\phi_0)=-\gamma/2\Omega$, and $\Omega=\sqrt{\omega_0^2-(\gamma/2)^2}$. These solutions are different with respect to the ones obtained for W_G, but the comparison of the exponential terms shows that similar effects may be expected for $\gamma=2\lambda\omega_0$.

The phase factor $\Phi(t)$ is

$$\Phi(t)=\frac{\omega_0 t}{2}+\frac{\gamma}{2\hbar}\int_0^t dt'\,v_{t'}^\gamma(u_{t'}^\gamma-u_{t'})+\frac{(v_t u_t^\gamma-u_t v_t^\gamma)}{\hbar} \tag{16}$$

and depends on γ. At large times when u and v become 0, the Gaussian wave packet becomes the ground state of H_0, and only Φ is keeping records on the past history of the wave function.

A general discussion of the multidimensional case may be found in [27].

B. Squeezed states

When $c_0\neq c_p$ and $u=0$ the wave packet represents a squeezed state, $|\psi_{sq}\rangle$ and it is nonstationary. Writing c_0 as $m\omega/\hbar$, we can easily see that the squeezed states are eigenstates for an oscillator Hamiltonian having the fre-

quency ω instead of ω_0. In the three-dimensional case, if H_0 corresponds to a spherical harmonic oscillator, then a squeezed state will be the ground state of a deformed harmonic oscillator. For the nuclear mean field, this change of shape appears when the quadrupole operator Q_0 has a nonvanishing ground state expected value and the quadrupole-quadrupole interaction term may be written as $-\delta m\omega_0^2(2z^2-x^2-y^2)/6$ with δ the deformation parameter. In this case, the oscillator frequencies are changed from ω_0 to $\omega_x=\omega_y=\omega_0\sqrt{1+\delta/3}$, and $\omega_z=\omega_0\sqrt{1-2\delta/3}$. In one dimension, the squeezed state $|\psi_{sq}\rangle$ is the eigenstate of the Hamiltonian $H_\mu=H_0-\mu K$ where $\mu=(\omega_0^2-\omega^2)/\omega_0$, $K=c_p x^2/2$, and it can also be related to the ground state of H_0 by a unitary transformation [28]

$$|\psi_{sq}\rangle=e^{-i\rho_0 S_2}|g\rangle\ , \tag{17}$$

with $\rho_0=\ln(\omega_0/\omega)$ and $S_2=i[(b^+)^2-b^2]/4=(xp+px)/4\hbar$.

Without dissipation this wave function becomes in time $e^{-(i/\hbar)H_0 t}|\psi_{sq}(0)\rangle$, and up to a phase factor it can be written as

$$|\psi_{sq}(t)\rangle=e^{-ip_t K}e^{-i\rho_t S_2}|g\rangle\ , \tag{18}$$

with

$$\rho_t=\ln\left[\frac{\omega_0}{\omega}\cos^2(\omega_0 t)+\frac{\omega}{\omega_0}\sin^2(\omega_0 t)\right]\ , \tag{19}$$

$$p_t=\frac{2}{c_p}\left[\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right]\frac{\sin(2\omega_0 t)}{8\sigma_t^2}\ , \tag{20}$$

and $\sigma_t^2=\langle x^2\rangle-\langle x\rangle^2=e^{\rho_t}/2c_p$. These equations show that the free squeezing motion may be observed as an oscillation in the width σ^2 of the wave packet having the frequency $2\omega_0$.

The Gisin dissipation term preserves the form (18) of the squeezed states, but has the effect of changing in time the frequency ω towards ω_0. Denoting this time-dependent frequency by $\omega(t)$, $\omega(t=0)=\omega$, then

$$\omega(t)=\omega_0\frac{e^{\rho_0}\tanh(\lambda\omega_0 t)+1}{e^{\rho_0}+\tanh(\lambda\omega_0 t)} \tag{21}$$

and the oscillations of the width are damped according to the law

$$\sigma_t^2=\frac{1}{2c_p}\left[\frac{\omega_0}{\omega(t)}\cos^2(\omega_0 t)+\frac{\omega(t)}{\omega_0}\sin^2(\omega_0 t)\right] \tag{22}$$

until the squeezed state becomes identical to the ground state $|g\rangle$.

The Albrecht procedure is ineffective; the squeezing oscillations are not damped by the addition of the term $W_A=\gamma x\langle p\rangle$ to H_0. According to (8), the energy is dissipated only if the momentum has a nonvanishing average value, which is not the case for the squeezed states. In fact, the squeezing represents an additional degree of freedom for the quantum dynamics, beside the "center of mass" degree of freedom and it is necessary to construct W_A using the appropriate coordinate and momentum operators. The comparison of the shifted states (11) and (17), which are ground states for H_{u_0} and H_μ, respectively, suggests that the relevant coordinate operator is related to the additional term in $H_{u_0(\mu)}$, responsible for the shift. For squeezing this operator is K, and the related momentum operator appears to be S_2. With this choice $i[H_0,K]/\hbar=2\omega_0 S_2$, similarly up to constant factors to $i[H_0,x]/\hbar=p/m$. It is worth noting also that the average $\langle\psi_{sq}|K|\psi_{sq}\rangle=c_p\sigma_t^2/2$ gives the width. However, by contrast to the usual commutation relations between the canonical coordinate and momentum $[x,p]=i\hbar$, the commutation relations between K and S_2 are more complicated, $[K,S_2]=iK$. Using these operators, we can write the corresponding Albrecht term for the squeezing dynamics as $W_{sq}=\gamma_{sq}K\langle S_2\rangle$.

In all the considerations about squeezing presented above, we have assumed that the coherent state motion is absent, such that $\langle x\rangle=0$ and $\langle p\rangle=0$. To account for this motion we have to write W_{sq} in a translation invariant form, and in the following this will be

$$W_{sq}=\frac{\gamma_{sq}}{2}(x-\langle x\rangle)^2(\langle xp+px\rangle-2\langle x\rangle\langle p\rangle)\ . \tag{23}$$

It is important to remark that this term has no effect on Eq. (5) and (6) for the center of mass motion. The dissipation law given by W_{sq} is

$$\frac{dE}{dt}=-\gamma_{sq}(\langle xp+px\rangle-2\langle x\rangle\langle p\rangle)^2 \tag{24}$$

and, as expected, it is proportional to the square of the average squeezing momentum.

IV. DISSIPATIVE TUNNELING THROUGH AN ASYMMETRIC DOUBLE-WELL POTENTIAL: NUMERICAL RESULTS

In the numerical calculations we have used the double-well polynomial potential V and false vacuum wave function ψ_0 (Fig. 1) employed previously [17,29] for the study of resonances in quantum mechanical tunneling. The potential is

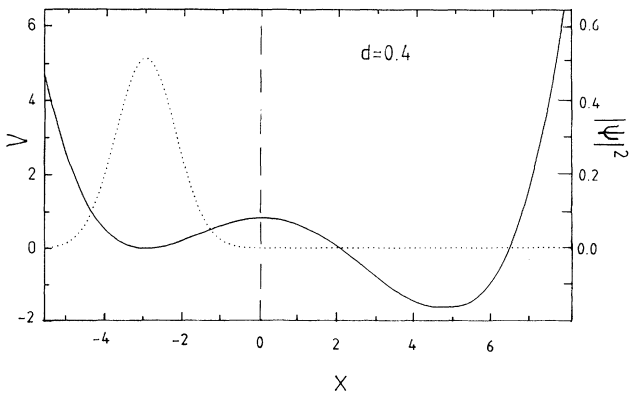

FIG. 1. The potential V (solid) and the initial wave function $|\psi_0|^2$ (dots).

$$V(x)=\begin{cases}\frac{5}{648}(x+3)^2(x-2)(x-6) & \text{if } x\le\frac{9}{2}\\ -\frac{625}{384} & \text{if } \frac{9}{2}\le x\le\frac{9}{2}+d\\ \frac{5}{648}(x-d+3)^2(x-d-2)(x-d-6) & \\ & \text{if } x\ge\frac{9}{2}+d\ ,\end{cases}$$

having the metastable minimum at $x=-3$ and the barrier height $V_M=V(x=0)=0.83$. Here, d is a parameter allowing a widening of the second well such that it may be tuned to obtain resonances. For a particle of unit mass, the first ones are obtained at $d=0.4$, 1.97, and 3.5, when avoided crossings between the energy levels of the Hamiltonian $H_0=p^2/2+V(x)$ occur [29]. The initial wave function ψ_0 is a Gaussian wave packet (10) having the parameters $u_0=-3$, $v_0=0$, and $\sigma_0^2=0.6$. This wave function is the ground state of a particle with $m=1$ in the harmonic oscillator potential having $\omega_0=0.69$, chosen to approximate $V(x)$ around the left-hand minimum. In the polynomial potential, the average energy $\langle\psi_0|H_0|\psi_0\rangle$ of ψ_0 is $E_0=0.425$. The bottom of the right-hand well, between $\frac{9}{2}$ and $\frac{9}{2}+d$ is -1.62 and at the first resonance ($d=0.4$) the true ground-state energy is -1.23.

The time-dependent Schrödinger equation for the Hamiltonian $H=H_0+W_{(t)}$, was solved numerically by the iterated leap-frog method [30], considering for $W_{(t)}$ the terms W_G, W_A, or $W_{\rm sq}$. The quantities of interest are the time-dependent probability $\rho(t)$ to find the wave function in the right-hand well ($x>0$) and the energy $E(t)$.

Without dissipation [$E(t)=E_0$] the tunneling ρ has an oscillatory behavior, represented in Fig. 2 for three values of d chosen such that one corresponds to the first resonance ($d=0.4$) and the other two are symmetric on both sides of the resonant value. At $d=0.4$, the energies of the second and third excited states of H_0 are $\epsilon_2=0.375$ and $\epsilon_3=0.400$, slightly lower than E_0. The quantum coherence oscillations [29,31] for this quasidegenerate doublet, $\rho_{2,3}(t)=\sin^2(\Delta t/2)$, $\Delta=|\epsilon_3-\epsilon_2|$, have maxima at odd multiples of $\pi/\Delta=125$, almost the same as of the free oscillation observed for our initial wave packet at $d=0.4$ (Fig. 2). This indicates that without dissipation, the second and third levels are indeed the relevant ones for the problem.

For $d<0.4$, one of these energies increase (ϵ_3), while the other maintains almost the same value and at $d>0.4$, one of the energies decrease (ϵ_2), while the other stays almost constant [29]. This behavior may be significant for the study of dissipative tunneling: If a loss of energy occurs, one may expect to observe an enhanced tunneling effect for $d=0.5$ as compared with $d=0.3$.

At low-to-moderate damping, for each of the terms W considered, one can distinguish three main stages. (1) For short times, the oscillatory behavior of ρ (existing in the absence of the dissipation) is still present, and for $d=0.4$, 0.5 the particle does not completely return in the first well. As expected, a different behavior is observed at $d=0.3$, when the dissipation has the tendency to localize the particle in the first well. The energy decrease during this stage is relatively small. These features are illustrated in Fig. 3(a) for W_A with $\gamma=0.05$. (2) When ρ approaches 0.5 a transitory stage starts, characterized by a strong decrease of energy and irregular oscillations of ρ. During this period, the wave function changes its shape fast to accommodate for the energy loss, becoming a combination of eigenstates localized in the right-hand well. (3) For large times ρ tends toward 1 and the energy becomes close to the ground-state energy. In this asymptotic stage the wave function is almost the ground-state Gaussian and it has damped oscillations in the second well by changing the average position and the width in a similar way as discussed in the previous section for the harmonic oscillator potential. For the Gisin model, the asymptotic stage is reached in a relatively short time, [see Fig. 3(b)], decreasing continuously when λ increases.

Beside the common qualitative features presented above, a more precise analysis reveals important differences between the three dissipative terms considered here. For a quantitative comparison it is useful to relate the time scale to a measurable quantity. Thus, one can try to compare the energy E for a given value of ρ, or ρ for a given amount of dissipated energy. An obvious choice for such reference values would be $\rho_{\rm ref}=1$ or $E_{\rm ref}$ equal to ground-state energy, but this is hard to use because, in general, such values are reached at very large computer times.

The calculations with the linear friction term W_A have shown that the main decrease of the energy appears after

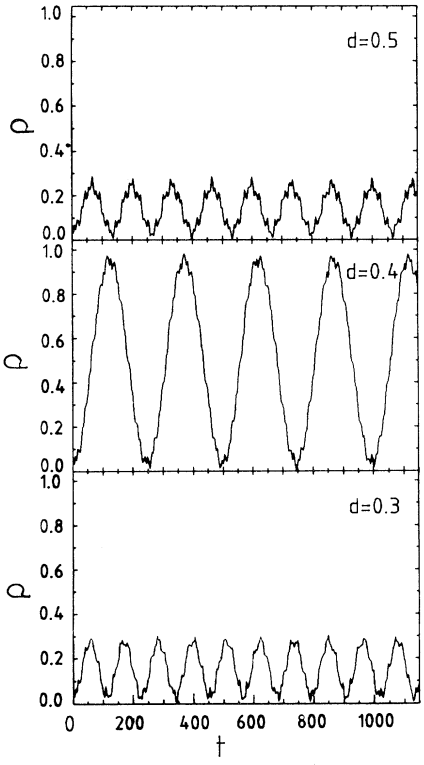

FIG. 2. The tunneling ρ without dissipation for $d=0.3$, 0.4, and 0.5.

the moment T_h of "half tunneling", when the localized probability in either well is the same for the first time, suggesting the choice $\rho_{\text{ref}}=0.5$. Another important moment is T_0, when the energy becomes negative, since the interval between T_h and T_0 gives an idea about the length of the transitory period.

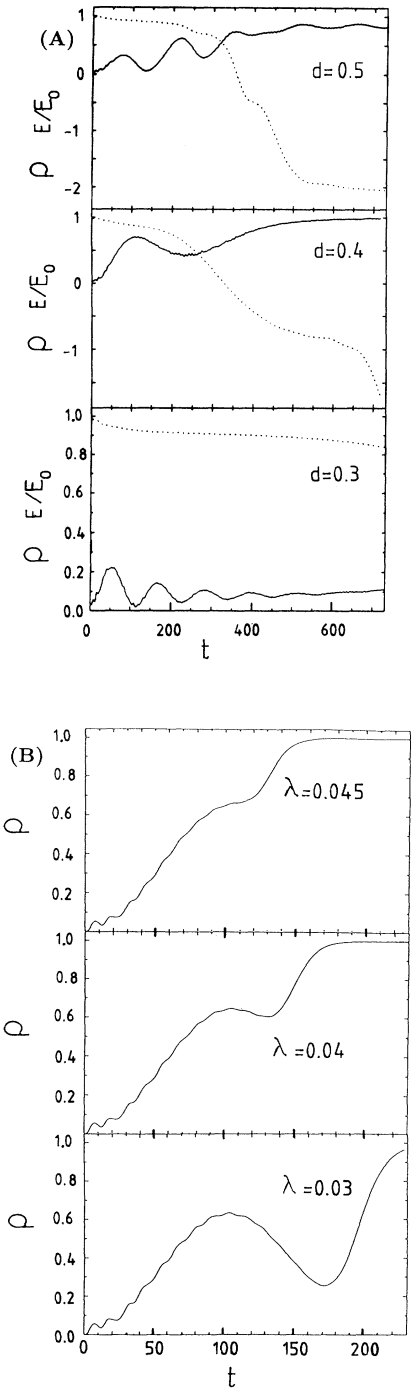

FIG. 3. (a) Linear damping. The time dependence of the tunneling ρ (solid) and the energy E/E_0 (dots) for $\gamma=0.05$. (b) Gisin damping. The time dependence of the tunneling ρ for $d=0.45$.

The half-tunneling time T_h and the corresponding value of the energy E_h are shown in Figs. 4–6 for the three damping terms investigated both in the resonant ($d=0.4$) and nonresonant case ($d=0.3$ and 0.5).

In the Gisin model (Fig. 4) for $d=0.3$ and 0.5, the tunneling of the first half of the wave function appears shortly after T_0 and is accompanied by the loss of a large amount of energy for all λ values, in a period of time monotonically decreasing with increasing λ. Without dissipation, for $d=0.3$ and 0.5, the tunneling is never 0.5 and this is why T_h increases very much for small λ. By contrast, for $d=0.4$ and $\lambda<\lambda_c$, $\lambda_c=0.075$, the tunneling is faster, less dissipative, and less dependent of λ. The maximum of ρ at $t=125$ is the last one surviving from the free quantum coherence oscillations, but it disappears at $\lambda=0.045$. If λ increases over λ_c, then T_h and E_h decrease strongly, following the behavior of the nonresonant case.

In the Albrecht model (Fig. 5), the first half of the wave function penetrates quickly without losing much energy (for $d=0.5$) or slowly losing a lot of energy (for $d=0.3$). The case $d=0.4$ shows both regimes depending on the value of γ. It is interesting to remark that when $\gamma=\gamma_c$, $\gamma_c=0.1$, the maximum of ρ at $t=125$ disappears and ρ remains almost constant (~ 0.5) during a relatively long interval $100\leq t\leq 220$. This value of γ would give self-trapping [13] in a symmetric double-well potential having the potential minima separated by the same distance $D=7.5$, as in the case of $V(x)$. The occurrence of this long-living state is reflected by the discontinuities of the E_h and T_h plots from Figs. 5(a) and 5(b). The ratio $\gamma_c/2\lambda_c=0.66$ is close to $\omega_0=0.69$, in agreement with the correspondence between γ and λ suggested by their damping effect on the coherent states, investigated in Sec. III A.

The results obtained with the squeezing dissipation, Eq. (23) are shown in Fig. 6. For $d=0.4$ and $\gamma_{\text{sq}}<0.005$, the dissipation effect is stronger than observed for W_G or W_A: a given amount of energy is lost in shorter time. This aspect shows that the squeezing mode is very important during the tunneling process. The change in the shape of the wave function means also a change in the width, and this is strongly affected by the squeezing dissipative term. At large γ_{sq}, the tunneling slows down and T_h increases, such as in the case of W_A, but E_h decreases and becomes negative as was observed for W_G.

Worth noting is that in Figs. 4(b) and 6(b) there is a unique value of the dissipation constant when $T_h=T_0$. For this value, the system arrives at half tunneling with 0 energy, by contrast to the linear friction case, Fig. 5(b), when the energy at T_h is always positive.

For linear friction and $\gamma=\gamma_c$, the values of T_h and T_0 are represented as function of d in Fig. 7. They do not have a monotonical behavior, but it appears to be an optimal d, higher than the resonant value, when the tunneling and the dissipation are favored such that T_h and T_0 have minima.

V. SUMMARY AND CONCLUSIONS

The topic of dissipative tunneling is one of the few related to such a wide variety of problems, ranging from

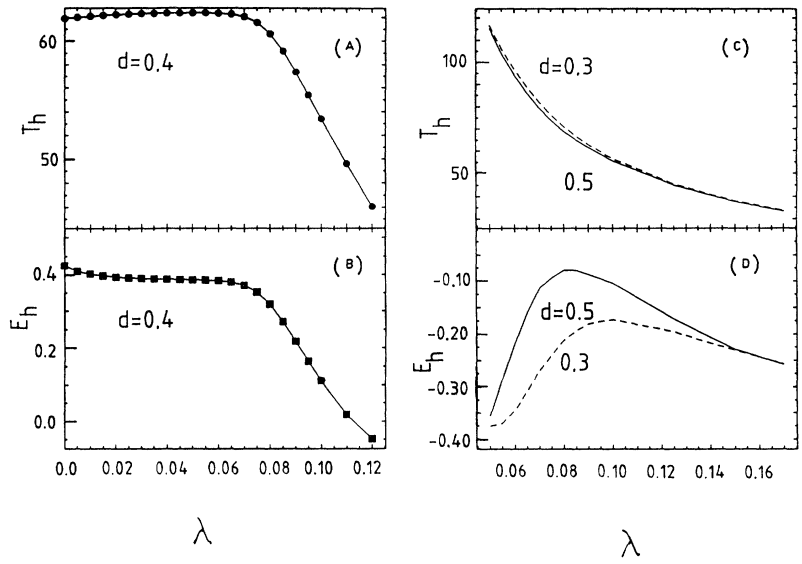

FIG. 4. Gisin damping. The λ dependence of the average energy (E_h) and time (T_h) at 50% tunneling: $d=0.4$ (a) and (b); $d=0.3$ (c) and (d), dash; $d=0.5$ (c) and (d), solid.

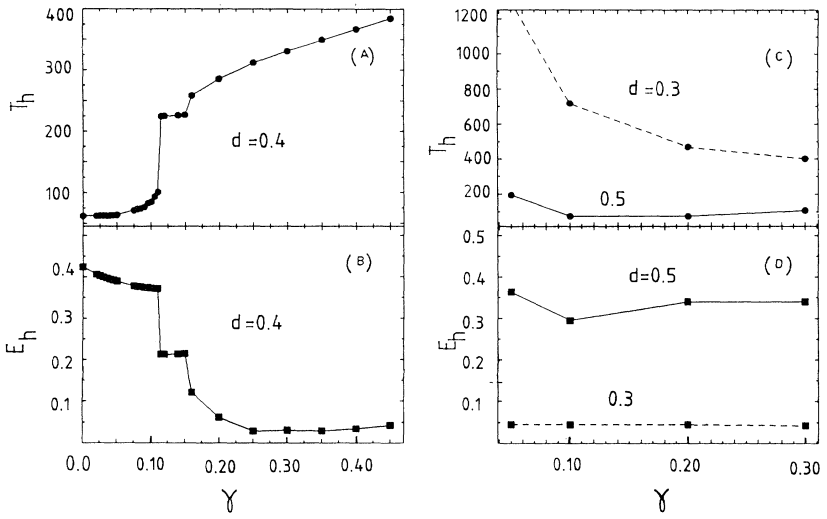

FIG. 5. Linear damping. The γ dependence of the average energy (E_h) and time (T_h) at 50% tunneling: $d=0.4$ (a) and (b); $d=0.3$ (c) and (d), dash; $d=0.5$ (c) and (d), solid.

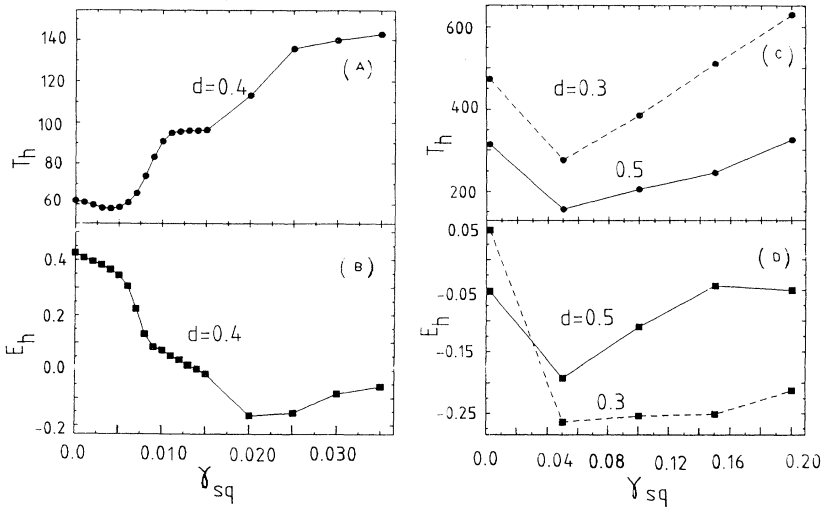

FIG. 6. Squeezing damping. The γ_{sq} dependence of the average energy (E_h) and time (T_h) at 50% tunneling: $d=0.4$ (a) and (b); $d=0.3$ (c) and (d), dash; $d=0.5$ (c) and (d), solid.

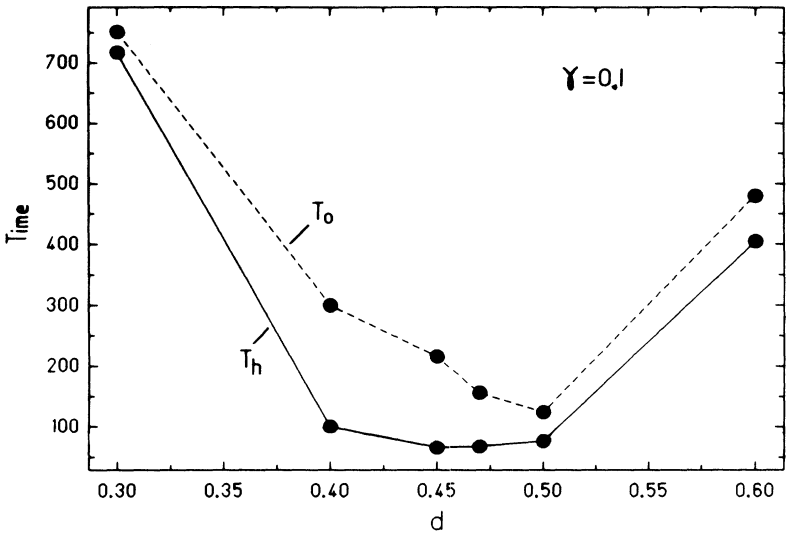

FIG. 7. Linear damping. T_h (solid) and T_0 (dash) as function of d for $\gamma=0.1$.

the conceptual foundations of quantum mechanics to the physics of Josephson junction or superconducting quantum interference devices. In particular, at the nuclear scale the dissipative phenomena play a key role in the understanding of fission or fussion reactions.

In this paper, the dissipation is treated phenomenologically, inserting nonlinear damping terms in TDSE. We consider three such terms, two employed before (W_G and W_A), and one (W_{sq}), constructed as a linear friction term for the squeezing motion of the quantum wave packet.

It is interesting to remark that the variational derivation of W_A proposed in the Appendix provides us naturally with an additional noise term. This term may induce "dipole"-like transitions between the stationary states, or it might become important for the treatment of a quantum Brownian particle. In general, the structure of the noise appearing in TDSE due to the coupling to the thermal bath was recently discussed [32] considering self-consistent terms that lead to a chaotic trajectory of the wave function in the Hilbert space. However, one should note that chaotic aspects were observed in the tunneling of the squeezed wave packets even without the bath coupling [23,31].

The three dissipative terms were compared first on a test system represented by a Gaussian wave packet in an arbitrary harmonic oscillator potential. Without dissipation this wave packet remains Gaussian and it has two oscillatory modes, one for the center of mass and one for the width. The analytical calculations have shown that a complete damping of both modes appears only for W_G. In this case, the center of mass oscillations have no frequency shift and are damped according to an exponential law, while the damping of the squeezing oscillations is more complicated [Eqs. (21) and (22)]. The term W_A produces a frequency shift for the center of mass oscillations and an exponential damping law, but proves to have no effect on the squeezing motion. Therefore, the complementary term W_{sq} (23) was constructed, damping completely the squeezing oscillation but having no effect on the center of mass motion.

The numerical analysis of the dissipative tunneling process in an asymmetric double-well potential has revealed important aspects that were not observed before. On short time the friction may have two opposite tendencies, depending on the resonance parameter d: if $d < d_{res}$ prevents the escape in the stable well, slowing down the process, or prevents the return to the metastable well of the wave function already escaped if $d \geq d_{res}$. Thus, near the resonance value d_{res}, d acts as a control parameter for the effect of dissipation on tunneling. A measure of this effect is represented by the change in the half-tunneling time T_h and of T_0 presented in Fig. 7. For linear friction, an interesting behavior was noticed at resonance, when γ becomes close to the self-trapping value: a quasistationary state appears, when ρ is almost constant (0.5) during a relatively large time interval, although the energy decreases.

At large times, ρ becomes very close to 1 for all models, but the energy goes to the ground-state value only in the Gisin case. For the other damping terms, the final state obtained was a Gaussian in the stable well performing low frequency oscillations of the average position and width. This residual motion is damped with a very slow rate, and a precise numerical investigation of the energy loss in the asymptotic limit becomes difficult.

The interplay between dissipation and resonance effects at tunneling presented above is the result of the exact calculations using TDSE and it cannot be obtained by a semiclassical treatment or within the two-level approximation. This is because the tunneling involves degrees of freedom of the wave function without a classical correspondent, while a consistent quantum description requires, in our case, at least three levels; the two in resonance and the true ground state. The present approach does not have such limitations and, therefore, is quite promising for further calculations with realistic potentials.

APPENDIX

Let us consider the joint system of a quantum particle and N classical harmonic oscillators (bath). If there is no

coupling, the time evolution of the oscillator coordinates q_i, $i=1,N$, and of the wave function ψ may be obtained from the variational equations

$$\delta_{q_i}\int L_{cl}(q_i,\dot{q}_i)dt=0$$

and

$$\delta_{\psi,\psi^*}\int L_q(\psi,\psi^*)dt=0\ , \tag{A1}$$

[33], where

$$L_{cl}=\sum_{i=1}^{N}(m_i\dot{q}_i^2-m_i\omega_i^2q_i^2)/2\ ,$$
$$L_q=\langle\psi|i\hbar\partial_t-H_0|\psi\rangle\ , \tag{A2}$$

and $H_0=p^2/2m+V(x)$ is the Hamiltonian operator of the isolated quantum particle.

If there is coupling the superposition principle is affected, and the joint system appears like a quantum system with superselection variables [34,35]. For this system, it is natural to assume that the dynamics will be given also by a variational equation

$$\delta_{(q_i,\psi,\psi^*)}\int(L_{cl}+L_q+L_{\rm coup})dt=0\ , \tag{A3}$$

with $L_{\rm coup}$ a coupling term depending on q_i, $i=1,N$, ψ,ψ^*, and eventually on their first time derivatives [36]. This variational equation may be written further in the form

$$\delta_{(q_i,\psi,\psi^*)}\int\left[\sum_{i=1}^{N}m_i(\dot{q}_i)^2+\langle\psi|i\hbar\partial_t-H|\psi\rangle\right]dt=0\ , \tag{A4}$$

where $H=H_0+H_b$, and H_b represents the bath energy plus the interaction operator. For bilinear coupling, this operator is

$$H_b=\sum_{i=1}^{N}\left[\frac{m_i\dot{q}_i^2}{2}+\frac{m_i\omega_i^2}{2}\left[q_i+\frac{C_i}{m_i\omega_i^2}x\right]^2\right]\ . \tag{A5}$$

Following the variational procedure, the equations of motion are

$$i\hbar\frac{\partial\psi}{\partial t}=\left[H_0+\frac{x^2}{2}\sum_{i=1}^{N}C_ig_i+x\sum_{i=1}^{N}C_iq_i\right]\psi\ , \tag{A6}$$

with $g_i=C_i/m_i\omega_i^2$ and

$$\dot{q}_i=\frac{p_i}{m_i}\ ,$$
$$\dot{p}_i=-m_i\omega_i^2q_i-C_i\langle\psi|x|\psi\rangle\ . \tag{A7}$$

The classical equations (A7) may be solved in terms of the unknown function of time $\langle\psi|x|\psi\rangle_{(t)}$, and their "retarded" solution is

$$q_i(t)=[q_i(0)+g_i\langle\psi|x|\psi\rangle_{(0)}]\cos(\omega_it)+\frac{\dot{q}_i(0)}{\omega_i}\sin(\omega_it)$$
$$-g_i\langle\psi|x|\psi\rangle_{(t)}$$
$$+g_i\int_0^t dt'\frac{d\langle\psi|x|\psi\rangle_{(t')}}{dt'}\cos[\omega_i(t-t')]\ . \tag{A8}$$

As expected, the classical bath oscillators are sensitive only to the average position and velocity of the quantum particle. Inserting this solution in (A6) we obtain a nonlinear Schrödinger equation $i\hbar\partial_t\psi=[H_0+W_{\rm int}(\psi)]\psi$ with

$$W_{\rm int}(\psi)=W_{\rm ren}-x\xi(t)+x\int_0^t\Gamma(t-t')\langle\psi|p|\psi\rangle_{(t')}dt'\ . \tag{A9}$$

Here

$$W_{\rm ren}=(x^2/2-x\langle\psi|x|\psi\rangle)\textstyle\sum_{i=1}^{N}C_ig_i\ ,$$
$$\xi(t)=-\sum_{i=1}^{N}\{C_i[q_i(0)+g_i\langle\psi|x|\psi\rangle_{(0)}]\cos\omega_it$$
$$+C_i\dot{q}_i(0)\sin(\omega_it)/\omega_i\}\ ,$$

and

$$\Gamma(t)=m^{-1}\textstyle\sum_{i=1}^{N}g_iC_i\cos\omega_it\ .$$

The first term $W_{\rm ren}$ can be considered as a renormalization potential in the intrinsic frame of the particle due to the coupling. It has no contribution to the right-hand side of the Ehrenfest Eqs. (5) and (6) for the center of mass motion and it will be neglected. For small N, $\xi(t)$ acts simply like an external driving force. If N is large and the bath coordinate and momenta at $t=0$ are statistically distributed, then $\xi(t)$ represents the noise and is related to $\Gamma(t)$ (the "memory function") by the fluctuation-dissipation theorem $\langle\langle\xi(t)\xi(s)\rangle\rangle=mkT\Gamma(t-s)$ [7], the brackets meaning statistical averaging. When the bath frequencies ω_i and the coupling constants C_i are such that Γ becomes proportional to a δ function, $\Gamma(t)=\gamma\delta(t)$, with γ the friction coefficient, then the last term in (A9) reduces to Albrecht's term. However, one should emphasize that it might be difficult to find a physical system satisfying all the peculiar conditions implied above.

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